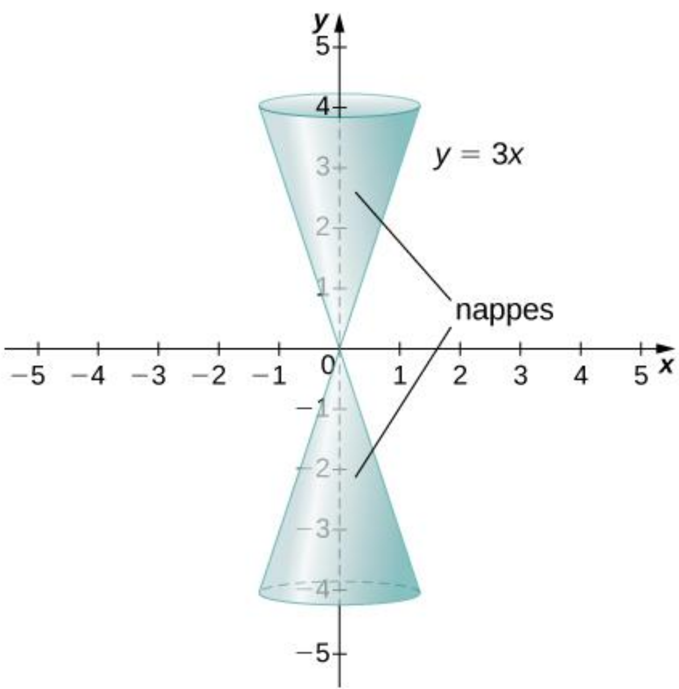
CONIC SECTION

CONE

DOUBLE NAPPED RIGHT CIRCULAR CONE

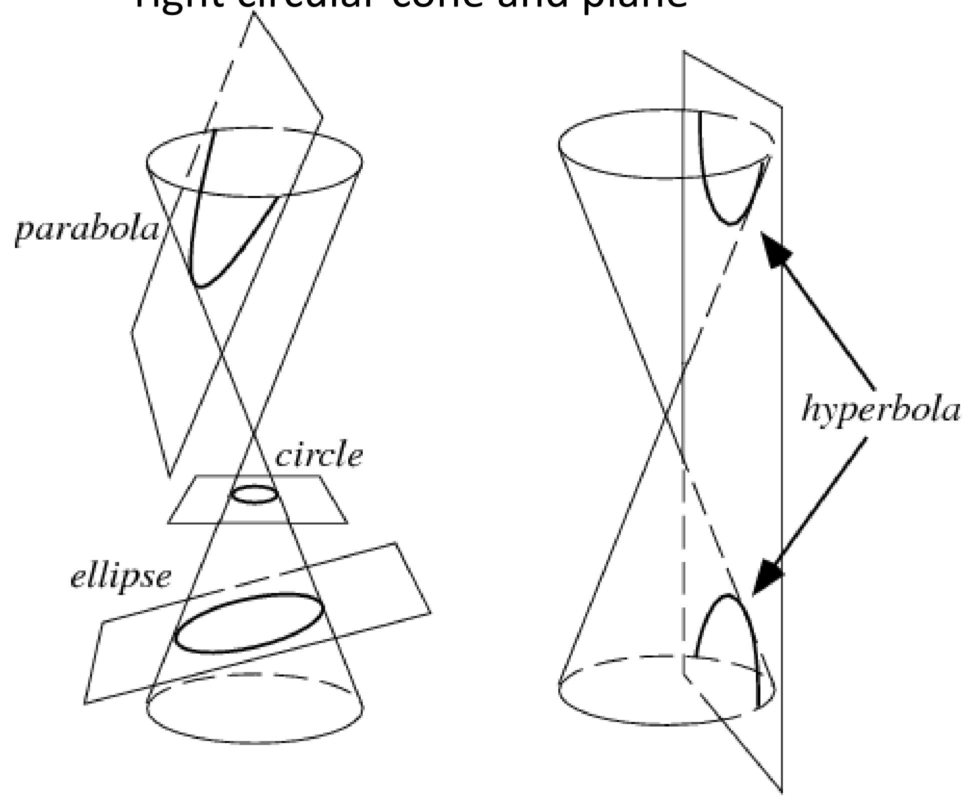
When two lines one vertical and y = mx intersect each other at a fixed point at an angle α. When line y = mx rotates about vertical line so that angle α remains same , double napped cone is formed



CONIC SECTION

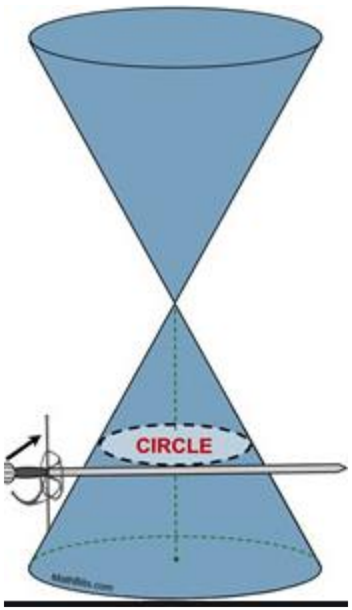
Conic sections are generated by the intersection of plane with a cone.

Figure formed by intersection of right circular cone and plane



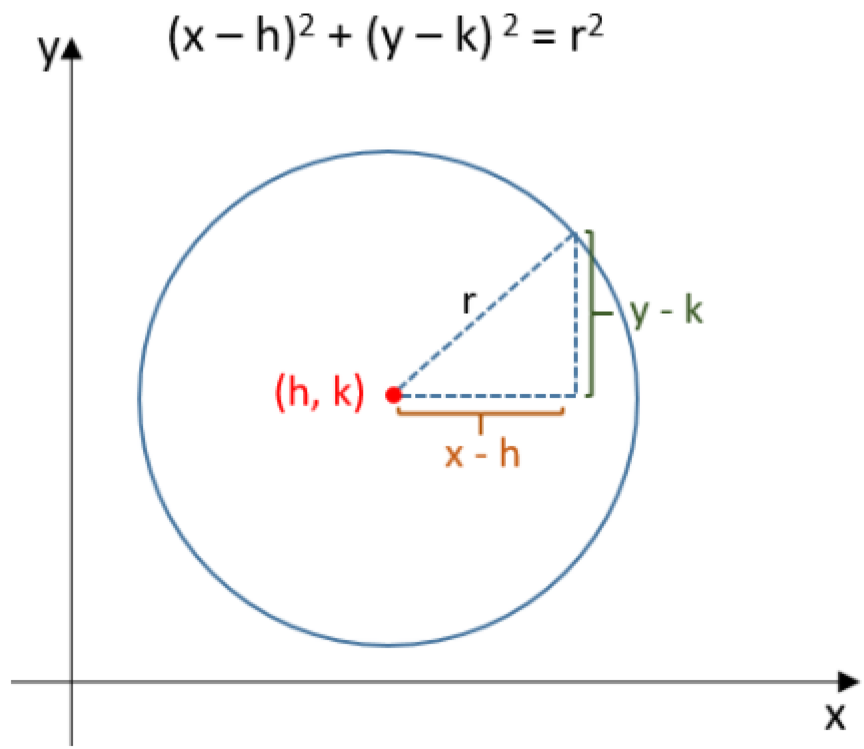
CIRCLE

If the plane is perpendicular to the axis of rotation then **circle** is formed



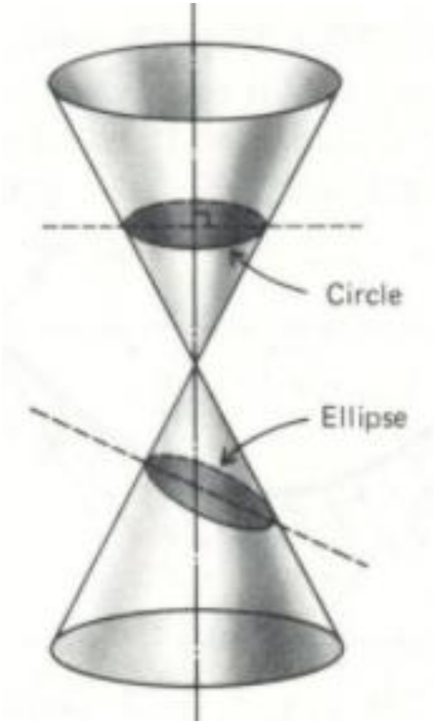
Here, angle of intersection of plane and cone, $\beta = 90^{o}$

Equation of circle centre at (h,k)



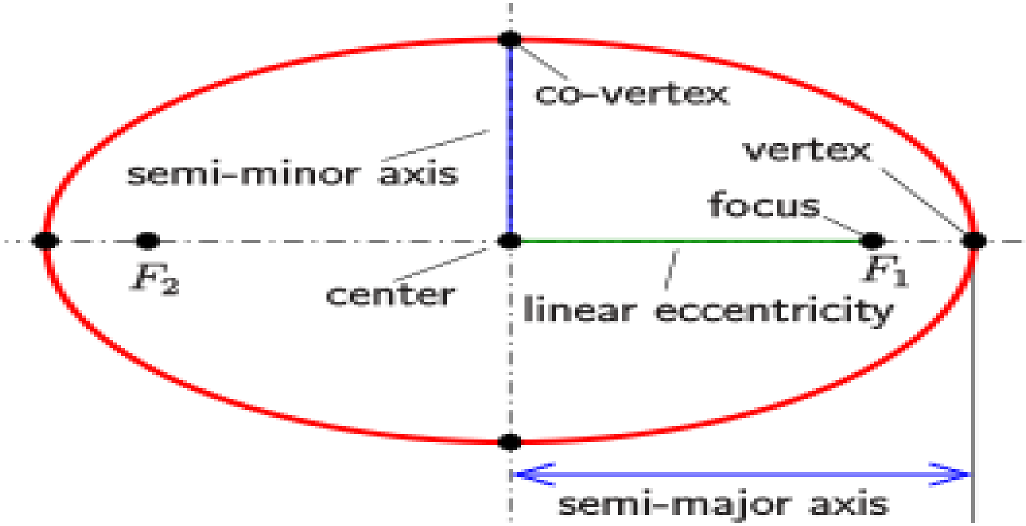
ELLIPSE

Intersection of a right circular cone and a plane that is not parallel to the base , the axis or an element of the cone



It can be observed that $\alpha<\beta<90^{o}$

So ellipse can be drawn as



Hence ellipse is a set of points for which sum of their distances from fixed points(foci) is constant

F(c,0)and $F^{|}$ (-c,0) are foci

$PP^{|}$ is **major axis which is longest distance across the ellipse**

P and $P^{|}$ are called vertices

co ordinates of P is (a,0) and $P^{|}$ is (-a,0)

$QQ^{|}$ is **minor axis which is smallest distance across the ellipse**

co ordinates of Q is (b,0) and $Q^{|}$ is (-b,0)

LENGTH OF MAJOR AXIS = 2a

LENGTH OF MINOR AXIS = 2b

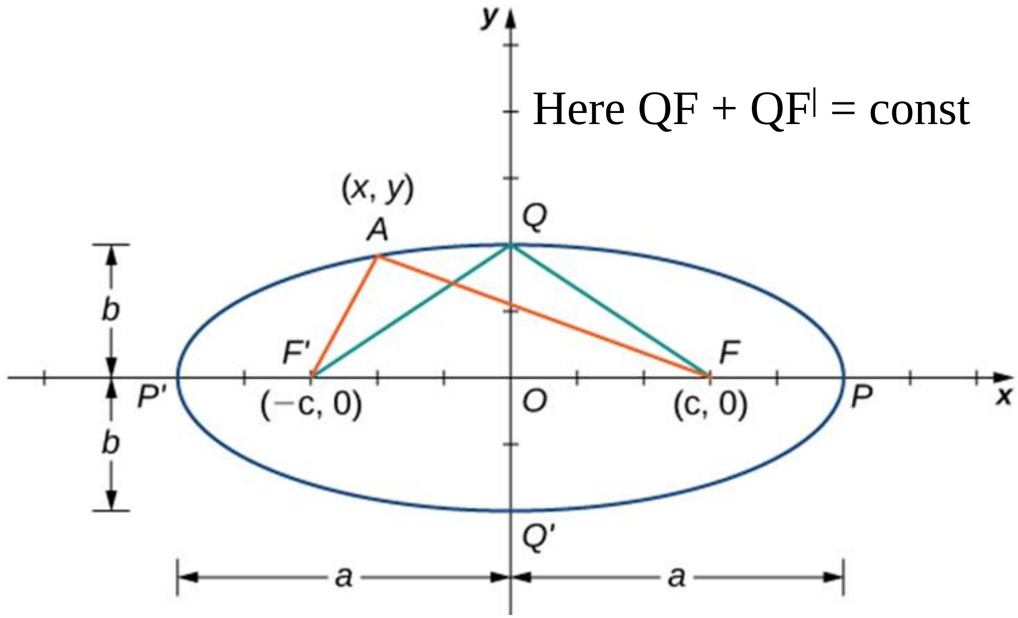
DISTANCE BETWEEN FOCI = 2c

Equation of ellipse is $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

eccentricity e =c/a < 1

Length of the Latus rectum = $\dfrac{2b^2}{a}$

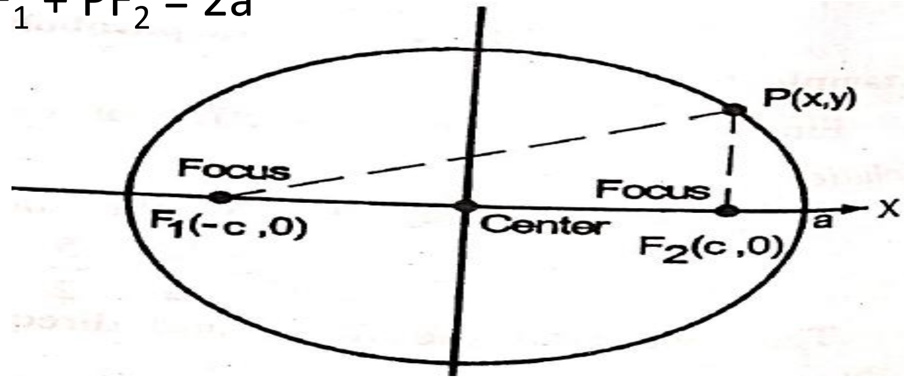
Here $QF + QF^{|}$ = const



Relation between a, b and c is $a^2+b^2=c^2$

Equation of ellipse

Suppose F_1 and F_2 are foci of ellipse then $PF_1 + PF_2 = 2a$



$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a.$$

$$\Rightarrow \sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$$

$$\Rightarrow (x+c)^2 + y^2 = [2a - \sqrt{(x-c)^2 + y^2}]^2$$

$$\Rightarrow x^2 + 2xc + c^2 + y^2$$

$$= 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + x^2 - 2xc + c^2 + y^2$$

$$\Rightarrow 4xc - 4a^2 = 4a\sqrt{(x-c)^2 + y^2}$$

$$\Rightarrow (xc - a^2)^2 = a^2[(x-c)^2 + y^2]$$

$$\Rightarrow x^2c^2 - 2xca^2 + a^4 = a^2x^2 - 2xca^2 + a^2c^2 + a^2y^2$$

$$\Rightarrow a^2(a^2 - c^2) = x^2(a^2 - c^2) + a^2y^2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1. \quad ....(1)$$

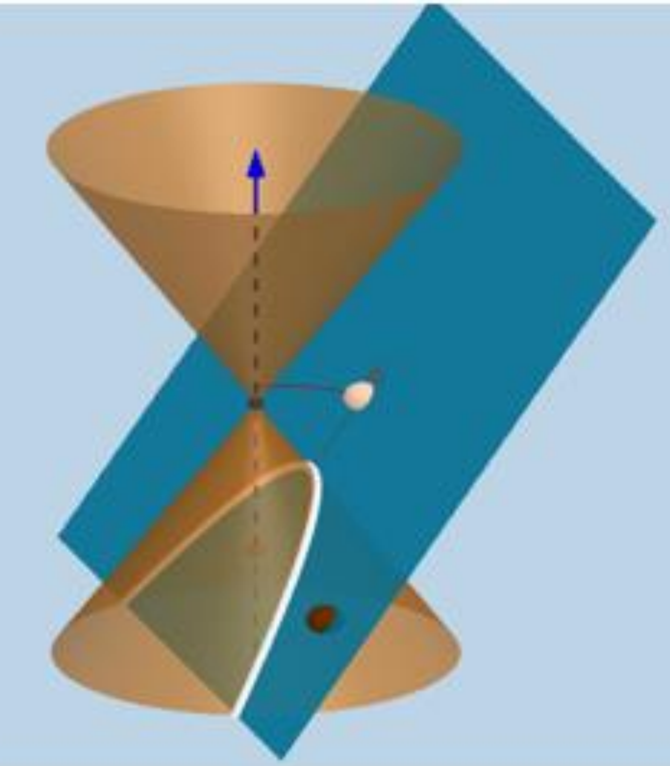
Since, $PF_1 + PF_2 > F_1 F_2 = 2a > 2c$ in triangle $PF_1 F_2$

Hence, $a > c$ so $a^2 - c^2 > 0$ in eq(1)

Suppose $b^2 = a^2 - c^2$ eq(1) becomes

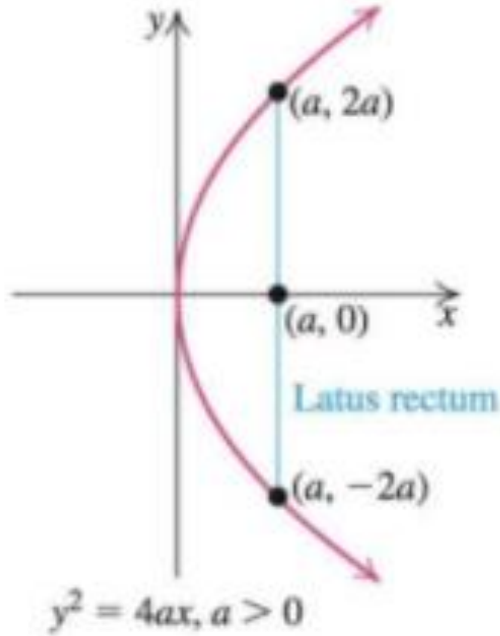
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

PARABOLA

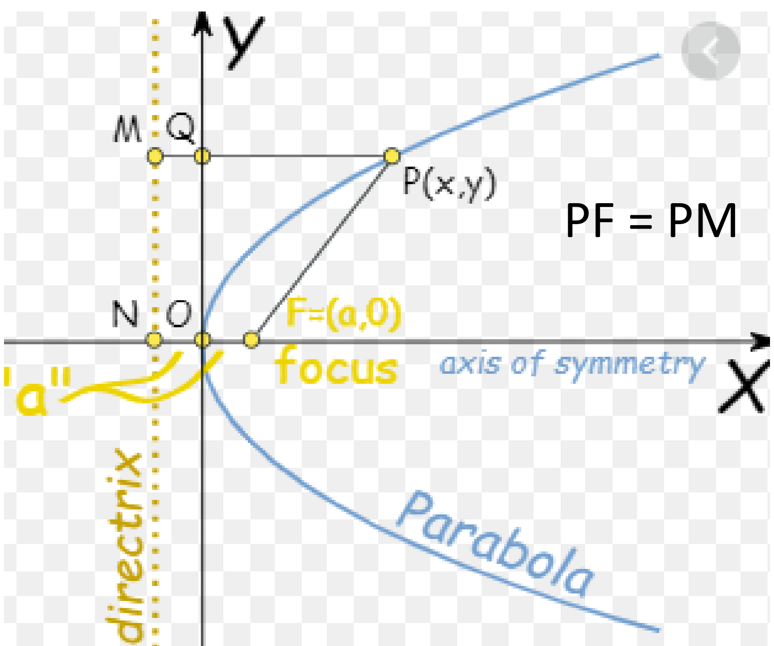


A **parabola** is a curve where any point is at an equal distance from: a fixed point (the focus), and. a fixed straight line (the directrix)

$\alpha = \beta$



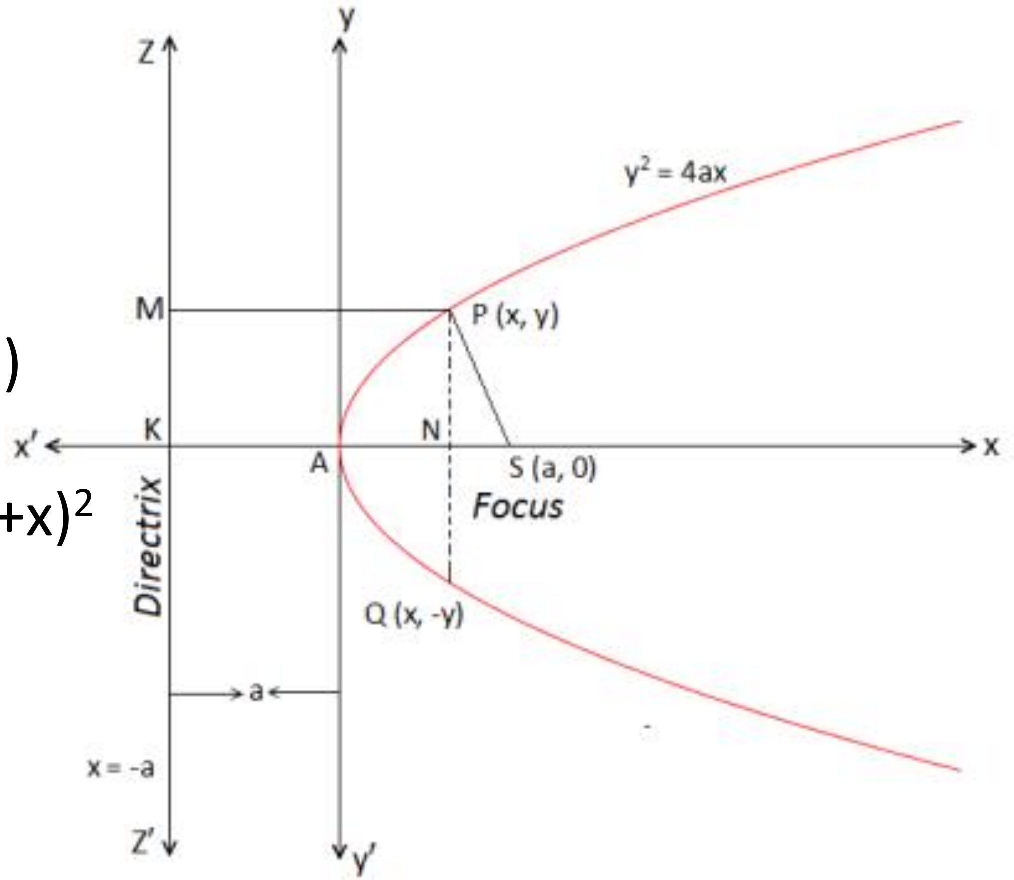
Latus rectum is a Latin word
Latus = side
Rectum = Straight



PF = PM

Latus rectum = 4a

EQUATION OF PARABOLA



Let $S(a,0)$ be the focus and $ZZ^{|}$ is directrix of the parabola.

Here $AS = AK = a,\ a > 0$

Since P lies on parabola

$\therefore$ $PM = PS \Rightarrow PM^2 = PS^2$ ....(1)

But $PM = NK = AK + AN$

$= a + x \Rightarrow PM^2 = (a+x)^2$

And $PS^2 = (x - a)^2 + y^2$

Using Equation (1), we get

$(a+x)^2 = (x - a)^2 + y^2$

Which on solving gives

$y^2 = 4ax.$

This equation is called

Standard equation of parabola

EXAMPLE

Find the focus and directrix of the parabola $y^2 = 10x$.

SOLUTION

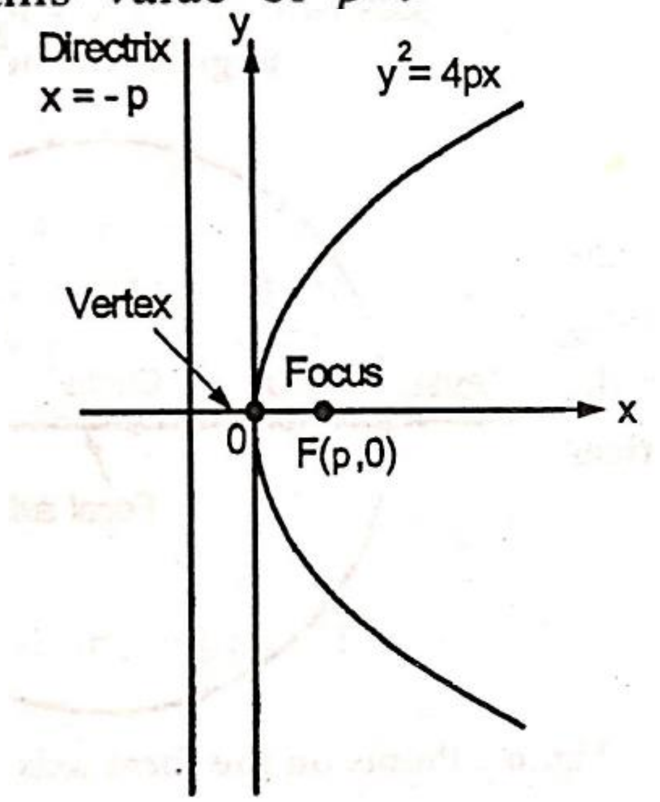
We find the value of p in the standard equation $y^2 = 4px$.

$$4p = 10, \quad \text{so} \quad p = \frac{10}{4} = \frac{5}{2}$$

Then we find the focus and directrix for this value of p :

Focus : $(p, 0) = \left(\frac{5}{2}, 0\right)$

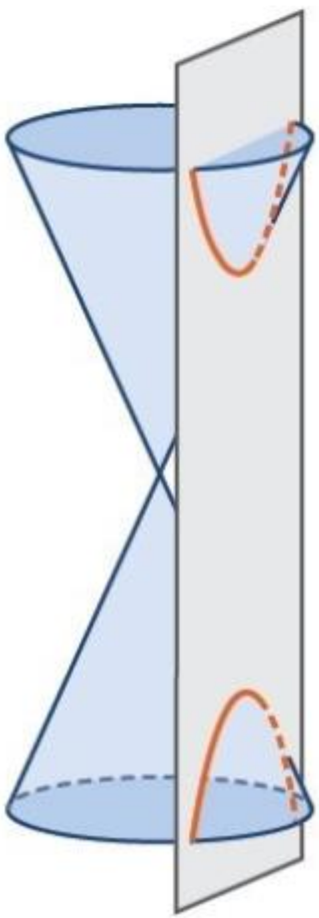
Directrix : $x = -p$ or $x = -\frac{5}{2}$

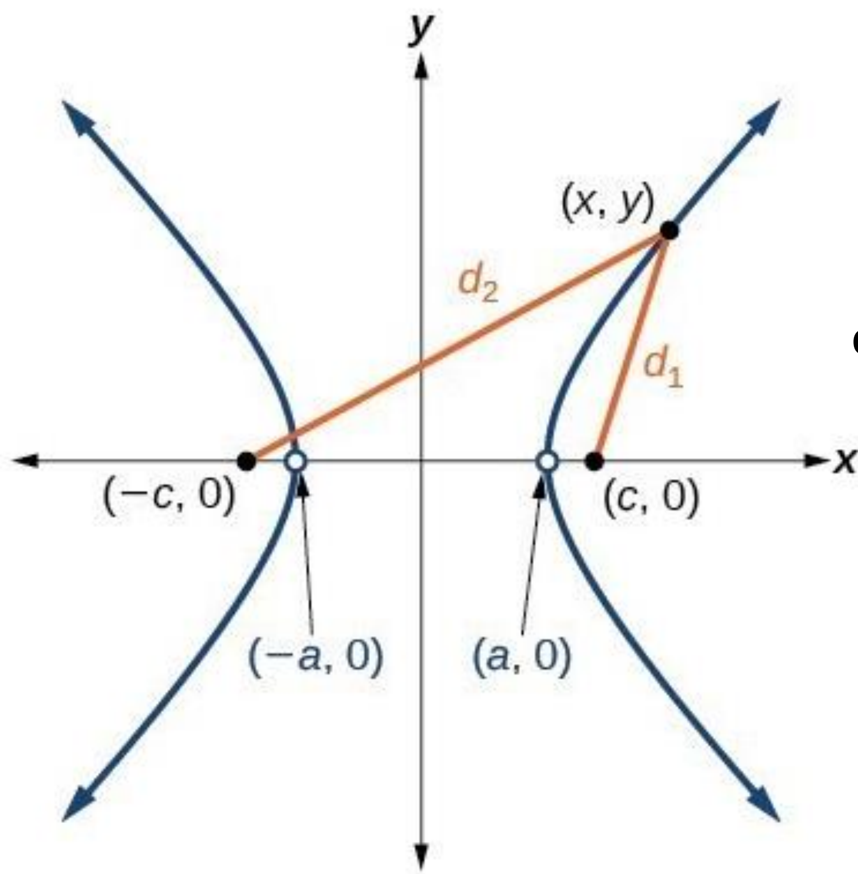


HYPERBOLA

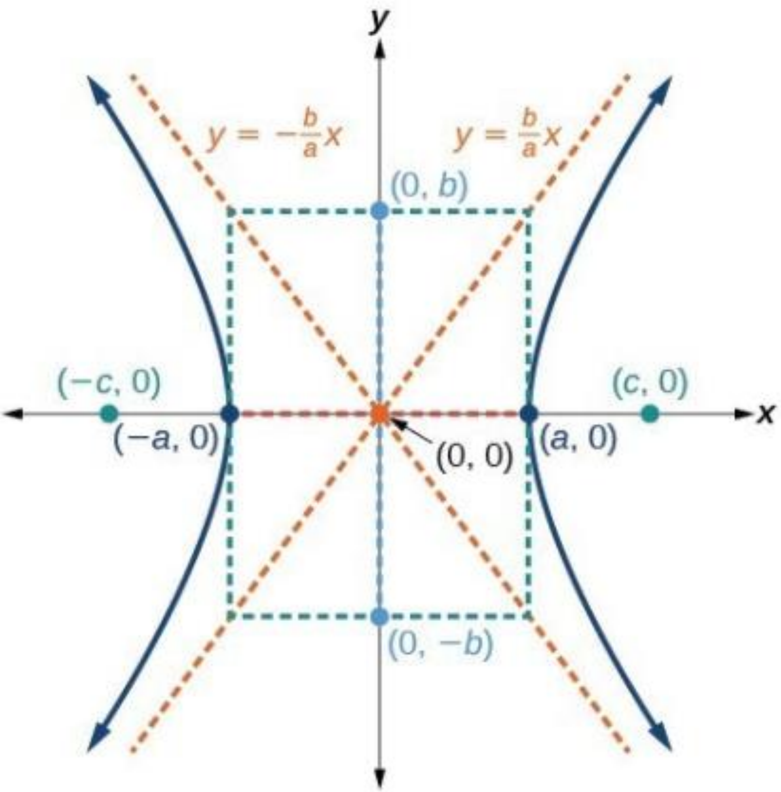
A symmetrical open curve formed by the intersection of a circular cone with a plane

Hyperbola is the set of all points in a plane, the difference of whose distances from fixed points is constant





$|d_2 - d_1|$ is constant



The standard form of the equation of a hyperbola with center (0,0) and transverse axis on the *x*-axis is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where
the length of the transverse axis is 2a
the coordinates of the vertices are (±a,0)
the length of the conjugate axis is 2b
the coordinates of the co-vertices
are (0,±b)
the distance between the foci is 2c
Where $c^2 = a^2 + b^2$
the coordinates of the foci are (±c,0)
the equations of the asymptotes are $y = \pm\frac{b}{a}x$

For hyperbola we have
Eccentricity e =c\a < 1 Because c > a

Equation of Hyperbola

Suppose co ordinates of foci F_1 and F_2 are (-c,0) and (c,0) respectively

By the definition of Hyperbola $PF_1 - PF_2 = 2a$ which on substituting values of PF_1 and PF_2 we get following

$$\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2} = \pm\, 2a$$

On solving above equation we get following

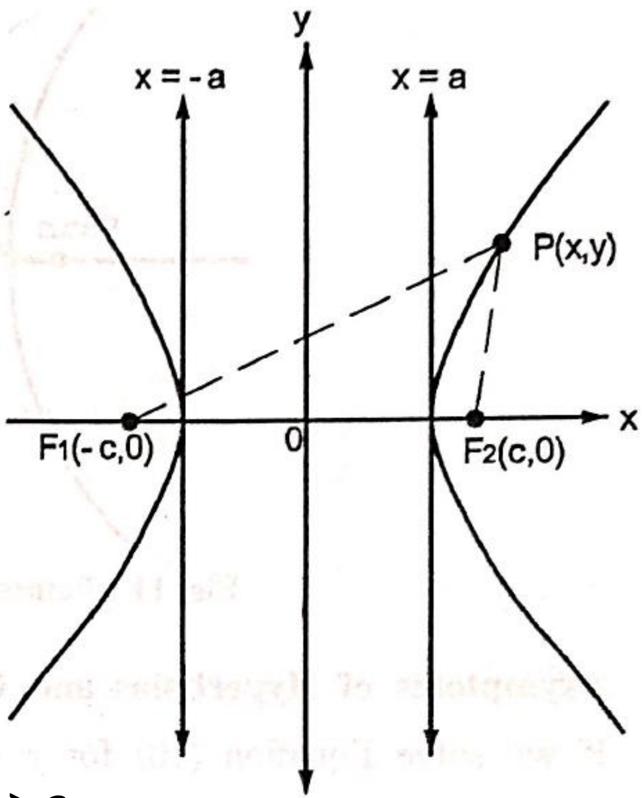
$$\frac{x^2}{a^2}+\frac{y^2}{a^2-c^2}=1 \qquad ....(1)$$

In triangle $PF_1\,F_2$, $F_1\,F_2 + P\,F_2 > PF_1$

therefore $PF_1 - PF_2 > F_1\,F_2$ Hence $2c > 2a$ which is $c > a$

Hence $c^2 - a^2 > 0$, using it eq(1) reduces to

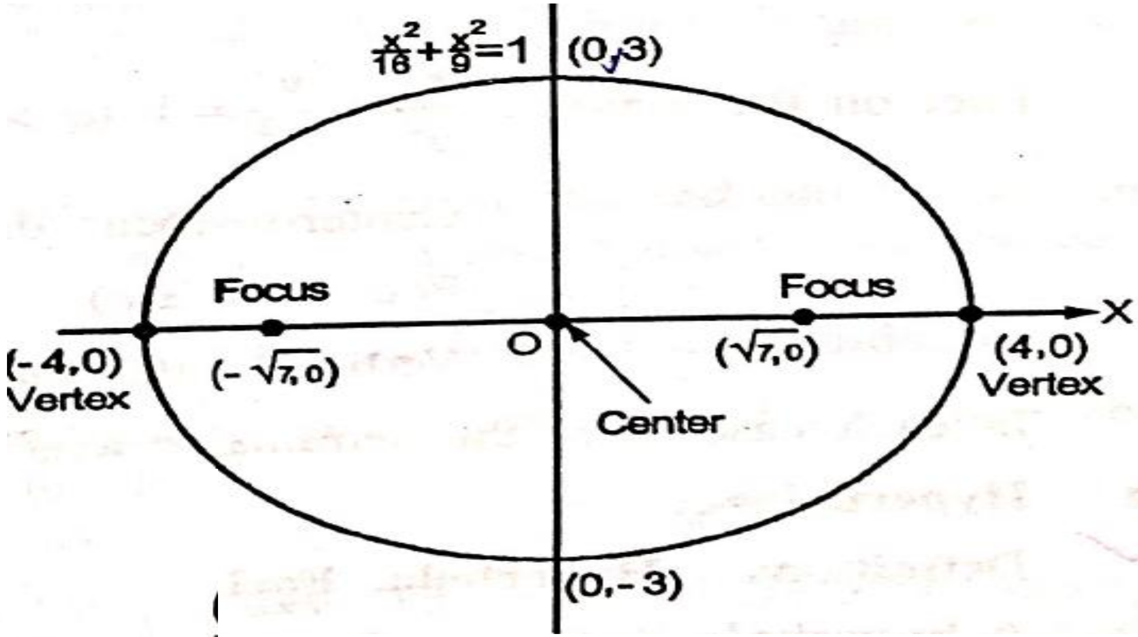
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \text{Where } b^2 = c^2 - a^2$$



EXAMPLE

Find semimajor axis, semiminor axis, center-to-focus distance, foci, vertices, and center for

(1) $$\frac{x^2}{16}+\frac{y^2}{9}=1$$



Semimajor axis : $a = \sqrt{16} = 4,$

Semiminor axis : $b = \sqrt{9} = 3$

Center-to-focus distance : $c = \sqrt{16-9} = \sqrt{7}$

Foci : $(\pm c, 0) = (\pm\sqrt{7}, 0)$

Vertices : $(\pm a, 0), (\pm 4, 0)$

Center : $(0, 0)$.

(2) $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

Semimajor axis : $a = \sqrt{16} = 4$,

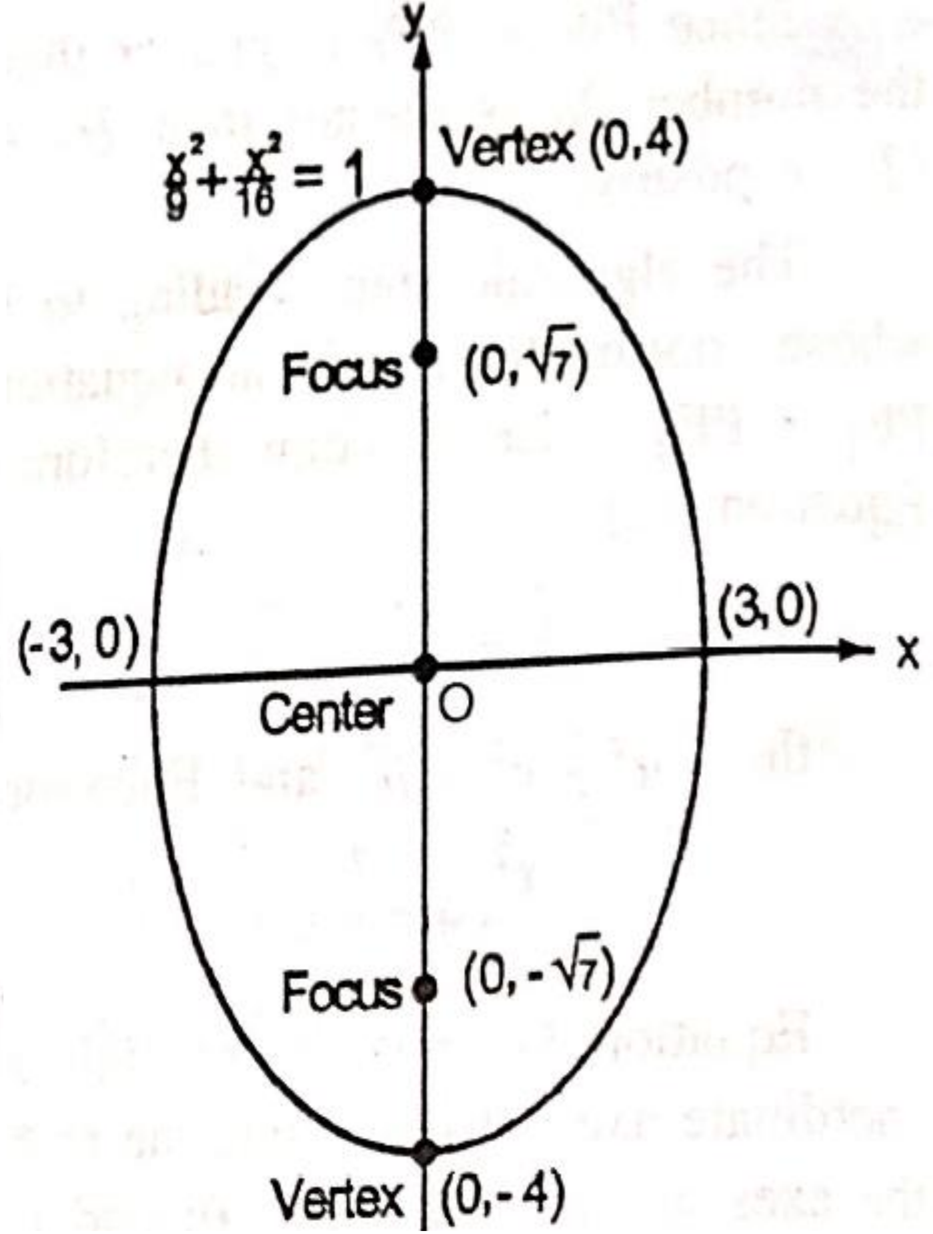
Semiminor axis : $b = \sqrt{9} = 3$

Center-to-focus distance : $c = \sqrt{16-9} = \sqrt{7}$

Foci : $(0, \pm c) = (0, \pm\sqrt{7})$

Vertices : $(0, \pm a)$, $(0, \pm 4)$

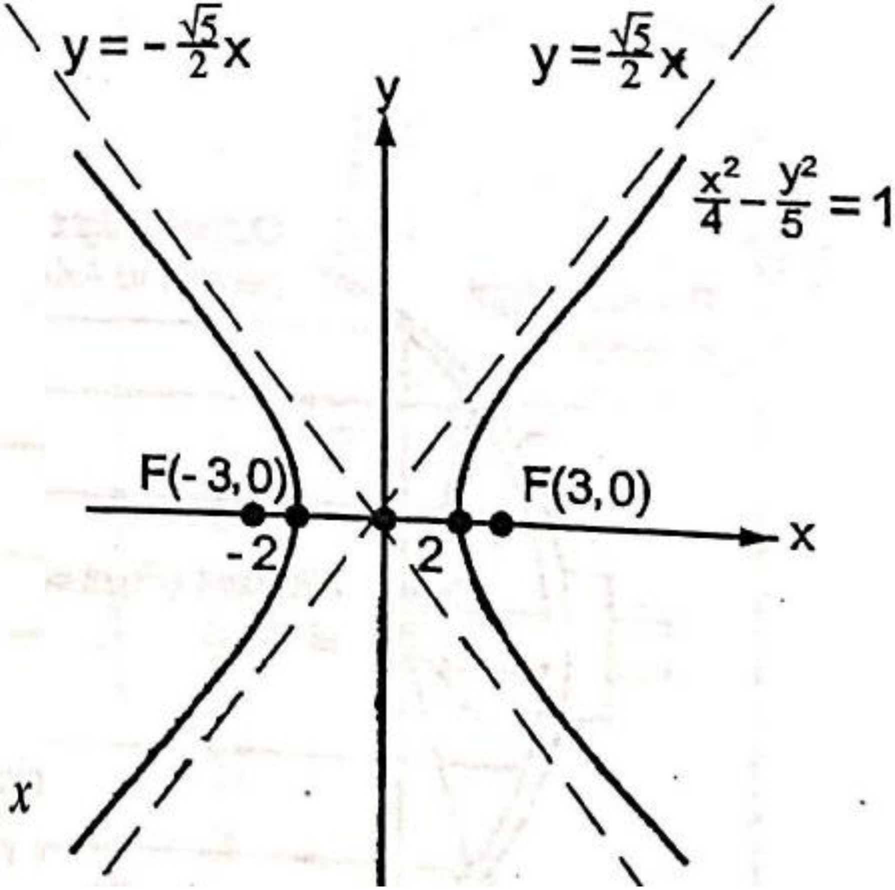
Center : $(0, 0)$.



EXAMPLE

Find center-to-focus distance, foci, vertices, center and asymptotoes for

(1) $$\frac{x^2}{4} - \frac{y^2}{5} = 1$$



Center-to-focus distance :

$$c = \sqrt{a^2 + b^2} = \sqrt{4+5} = 3$$

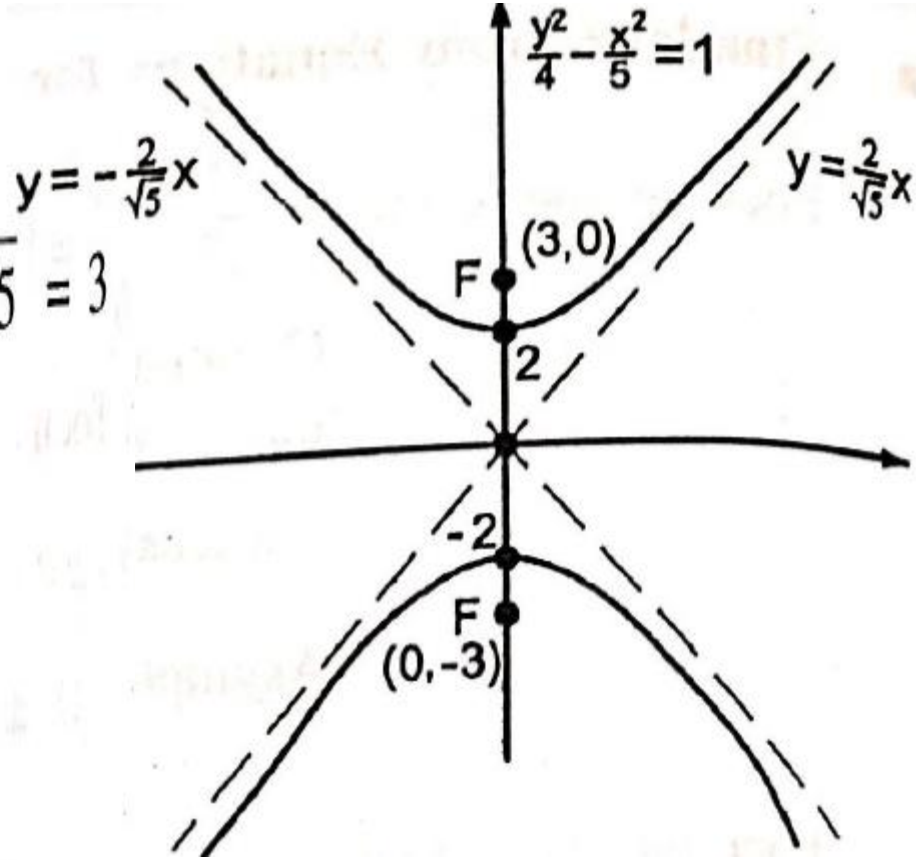
Foci : $(\pm c,\ 0) = (\pm 3,\ 0)$

Vertices : $(\pm a,\ 0)$, $(\pm 2,\ 0)$

Center : $(0,\ 0)$.

Asymptotes : $\frac{x^2}{4} - \frac{y^2}{5} = 0$ or $y = \pm\frac{\sqrt{5}}{2}x$

(2) $\dfrac{y^2}{4} - \dfrac{x^2}{5} = 1$



Center-to-focus distance : $c = \sqrt{a^2 + b^2} = \sqrt{4+5} = 3$

Foci : $(0, \pm c) = (0, \pm 3)$

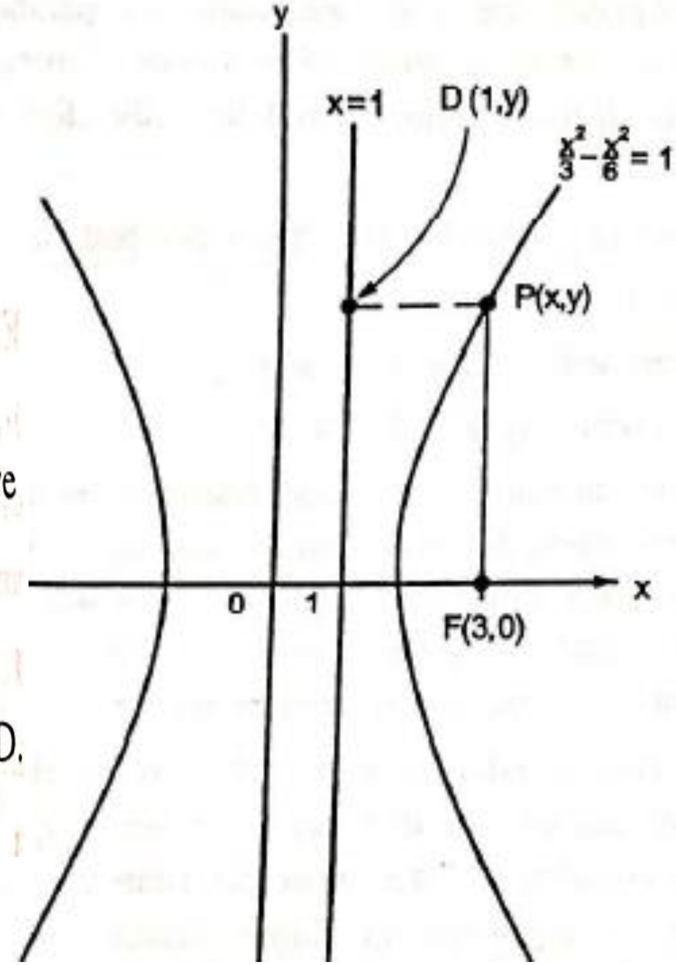
Vertices : $(0, \pm a) = (0, \pm 2)$

Center : $(0, 0)$.

Asymptotes : $\dfrac{y^2}{4} - \dfrac{x^2}{5} = 0$ or $y = \pm \dfrac{2}{\sqrt{5}}\, x$

EXAMPLE

Find a Cartesian equation for the hyperbola centered at the origin that has a focus $(3, 0)$ and the line $x = 1$ as the corresponding directrix.



$$(c, 0) = (3, 0) \quad \text{so} \quad c = 3,$$

The directrix is the line

$$x = \frac{a}{e} = 1, \quad \text{so} \quad a = e.$$

When combined with the equation $e = c/a$ that defines eccentricity, these results give

$$e = \frac{c}{a} = \frac{3}{e}, \quad \text{so} \quad e^2 = 3 \text{ and } e = \sqrt{3}$$

Knowing e, we can now derive the equation we want from the equation $PF = e \cdot PD$.

$$\sqrt{(x-3)^2 + (y-0)^2} = \sqrt{3}\,|x-1| \qquad (e = \sqrt{3})$$

$$x^2 - 6x + 9 + y^2 = 3(x^2 - 2x + 1)$$

$$2x^2 - y^2 = 6$$

$$\frac{x^2}{3} - \frac{y^2}{6} = 1.$$